

Quantifier Truth Trees: Solved Examples

Order of Use for Truth Tree Rules:

1. **Non-branching sentence logic rules** (True and False Negation, True Conjunction, False Disjunction)
2. **Check rules for quantifiers** (True Existential, False Universal)
3. **Star rules for quantifiers** (True Universal, False Existential)

Example 1. $\sim\forall x Gx \therefore \exists x \sim Gx$

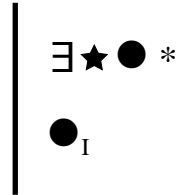
$$\begin{array}{c|c} \sim\forall x Gx & \exists x \sim Gx \end{array}$$

First break down the true negation “ $\sim\forall x Gx$ ”.

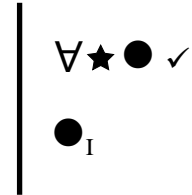
$$\begin{array}{c|c} \checkmark \sim\forall x Gx & \begin{array}{c} \exists x \sim Gx \\ \forall x Gx \end{array} \end{array}$$

Now we're faced with the false existential " $\exists x \sim Gx$ " and the false universal " $\forall x Gx$ ". **False Universal is a star rule**, while **False Existential is a check rule**.

False Existential

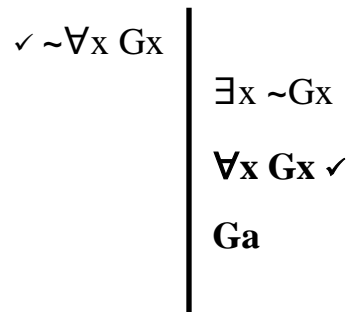


False Universal



(Restriction: \bullet_I must use a **new** name letter)

So we **break down the false universal** " $\forall x Gx$ " **first**, instantiating to a **new name letter** (that is: a name letter which hasn't appeared in the tree yet). Since no name letters appear in the tree so far, we're free to instantiate the false universal to any name letter – here, "**a**".



Next we break down the false existential " $\exists x \sim Gx$ ". There are no constraints on what name can be used with the False Existential rule. And as is our practice generally with star rules, we instantiate only to name letters already appearing on the tree.

Hence we **instantiate** “ $\exists x \sim Gx$ ” to name letter “a”.

False Existential

$\exists \star \bullet *$ \bullet_I	
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$\checkmark \sim \forall x Gx$

	$\exists x \sim Gx *$
--	-----------------------

	$\forall x Gx \checkmark$
--	---------------------------

	Ga
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	$\sim Ga$
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Finally we break down the false negation “ $\sim Ga$ ” with the False Negation rule.

$\checkmark \sim \forall x Gx$	
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	$\exists x \sim Gx *$
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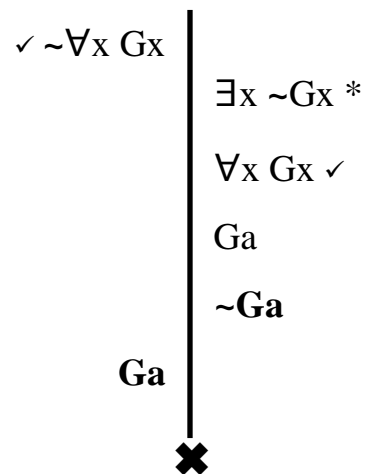
	$\forall x Gx \checkmark$
--	---------------------------

	Ga
--	----

	$\sim Ga$
--	-----------------------------

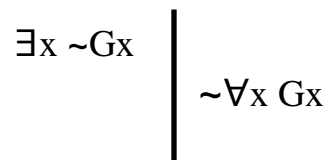
Ga	
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Since “Ga” is both true and false here, **the tree path closes**.

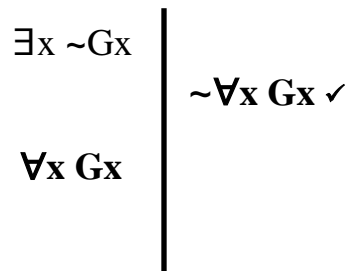


Hence the argument “ $\sim \forall x Gx \therefore \exists x \sim Gx$ ” is **valid**. (Since this is one of the Quantifier Negation rules, this is no surprise.)

Example 2. $\exists x \sim Gx \therefore \sim \forall x Gx$

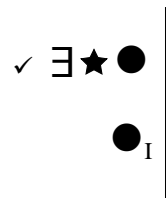


We first dispatch the false negation.

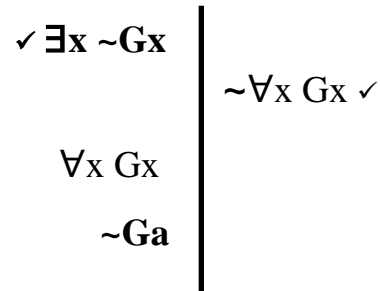


Faced with a true existential and a true universal, we first break down the true existential – because the **True Existential** rule is a **check rule**, taking priority over the star rule True Universal. We instantiate to new name letter “a”.

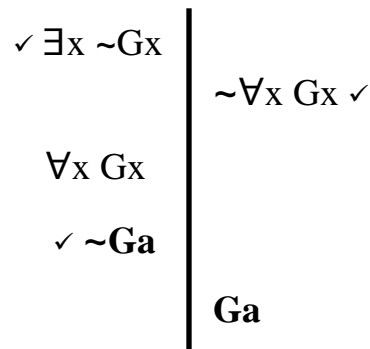
True Existential



(Restriction: \bullet_I must use a **new** name letter)

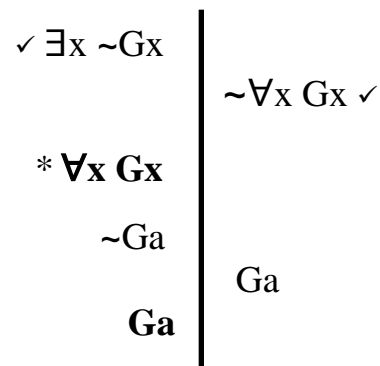
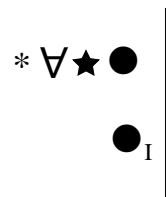


We next clear out the true negation “ $\sim Ga$ ”.



Finally we instantiate “ $\forall x Gx$ ” to “ Ga ” by the **star rule True Universal**.

True Universal



Since “Ga” is both true and false on this path, **the tree closes**. The **argument** “ $\exists x \sim Gx \therefore \sim \forall x Gx$ ” is **valid**.

